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TECHNICAL NOTE

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METHOD FOR EVALUATING FROM SHADOW OR SCHLIEREN PHOTOGRAPHS THE PRESSURE DRAG IN TWO-DIMENSIONAL OR AXIALLY SYMMETRICAL FLOW PHENOMENA WITH DETACHED SHOCK

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SUMMARY

A method has been developed for evaluating from shadow or schlieren photographs the pressure drag of axially symmetrical bodies at zero angle of attack or of two-dimensional bodies producing detached shock. The method consists in the determination of the flow properties along a characteristic line and the application of the momentum theorem to a stream tube around the body. The method can be applied to the determination of external drag of supersonic inlets with subsonic flow at the entrance of the inlet.

INTRODUCTION

The system of characteristics permits the pressure drag and the shape of the shock for axially symmetrical and two-dimensional flow phenomena having everywhere supersonic speed to be determined directly from the boundary conditions. In this case if viscous effects are neglected, the value of the pressure drag can be determined analytically, and no experimental measurement is necessary. The determination of the shape of the shock can be useful in making a comparison between experimental and theoretical results and permits an evaluation of the displacement thickness of the wake and of the boundary layer along the body. The characteristic system does not permit, however, direct determination of the pressure drag when the shock is detached from the body, because it cannot be applied to the subsonic part of the flow.

The determination of the pressure drag of bodies producing detached shocks is important in many problems; for example, the bodies of revolution used in practical applications often have round noses and, in supersonic inlets, conditions exist in which the flow at the entrance of the inlet is subsonic. The experimental measurement of the pressure drag for these cases usually requires complicated experimental equipment; therefore, the determination of drag from shadow or schlieren photographs of the phenomena can be very useful for practical applications. A method is presented herein that permits determination of the pressure drag directly from shadow or schlieren photographs. This system can be

especially useful in all cases in which the experimental technique uses the principle of moving models (shooting ranges, whirling arms, etc.) and in experiments of supersonic inlets in which also the internal flow produces forces and in which it is, therefore, difficult to measure the external drag independently of the internal drag. The method gives the value of the pressure drag due to the subsonic and supersonic regions of the flow and permits also the determination of pressure and velocity distribution of the supersonic region, from which the shape and thickness of the wake can be obtained.

SYMBOLS

x, y Cartesian coordinates

n normal to streamline

V velocity

V_l limiting velocity

W velocity in terms of the limiting velocity (V/V_l)

M Mach number

ρ density

p pressure

s entropy

m_A, m_B mass flow

$l = \dfrac{\sin\beta \tan\beta \sin\varphi}{\cos(\varphi + \beta)}$ for axially symmetrical phenomena; $l = 0$ for two-dimensional phenomena

$m = \dfrac{\sin\beta \tan\beta \sin\varphi}{\cos(\varphi - \beta)}$ for axially symmetrical phenomena; $m = 0$ for two-dimensional phenomena

φ inclination of the velocity vector with respect to x-axis

ϵ inclination of the shock with respect to upstream velocity

β Mach angle $\left(\sin^{-1}\frac{1}{M}\right)$

c_v specific heat at constant volume

γ ratio of specific heats

R gas constant

Subscripts:

o refers to stagnation free-stream conditions

1 refers to free-stream properties

A points of first family

B points of second family

C quantities at the points calculated from A and B

P quantities at any point P behind the shock

THE METHOD FOR DETERMINING THE PRESSURE DRAG

In any supersonic phenomenon in which a detached shock exists, the flow is subsonic only in a small zone of the space and the speed outside of this zone, which is usually limited to the vicinity of the nose of the body, becomes again supersonic. The body considered has finite dimensions. In the supersonic part of the flow the characteristic theory can be applied to determine the pressure drag and the pressure distributions. The line HN of figure 1 is the sonic line that divides the subsonic from the supersonic region, and from any point G of the shock in the supersonic region the characteristic line GD of the second family that meets the body at some point D (see, for example, reference 1) can be determined.

If the flow properties are known along the line GD, the component of the momentum of the stream along GD in the direction of the free-stream velocity and the resultant of the pressure on the surface GD in the same direction can be evaluated. The resultant of the pressure along the surface OD in the direction of the x-axis can be obtained from the momentum law. Indeed, the difference of momentum between the surfaces EE' and GD must be equal to the resultant of pressures at the surfaces E'G, EE', EO, OD, and GD. The pressure along E'G and EO does not produce any component of force in the direction of the x-axis. The resultant of pressure at the surfaces EE' and GD is known (along EE' the pressure is equal to the free-stream pressure); therefore, the resultant of the pressure in the direction of the x-axis along OD can be evaluated from the difference of momentum in the direction of the x-axis between EE' and GD. When supersonic inlets having subsonic entrance velocity are considered (fig. 2), the streamline EO is not

parallel to the stream direction and, therefore, with this system the pressure drag along this line is also determined.

In these considerations the viscous effects have been neglected, but the viscous effects are small in the front part of the body and therefore can be considered independently. For the back part of the body the considerations made in the "Introduction" on the possibility of the determination of the displacement thickness of the boundary layer and of the wake are still valid. It should be noted that, by consideration of the change in momentum in the y direction, the lift of two-dimensional bodies at an angle of attack can be determined. From the preceding considerations the determination of the pressure drag along EOD (fig. 1) is reduced to the determination of the flow properties along GD. The flow properties along GD can be determined from the shape of the shock in the following way:

If EGL is the shape of the shock determined from experimental measurements (shadow or schlieren photographs), the inclination of the shock along GL (fig. 1) can be measured. In order to obtain the inclination of the shock with some precision the coordinates of the shock can be measured and an analytical expression can be determined for the curve.

From the inclination of the shock ϵ, the magnitude W and the direction φ of velocity and the variation of entropy Δs across the shock can be calculated at any point P of the shock, for example, from the equations:

$$\frac{1}{\tan \varphi_P} = \left(\frac{\gamma + 1}{2} \frac{{M_1}^2}{{M_1}^2 \sin^2\epsilon - 1} - 1\right) \tan \epsilon \tag{1}$$

$$\frac{\tan \epsilon_P}{\tan(\epsilon - \varphi)_P} = \frac{\gamma - 1}{\gamma + 1} \left[\frac{1 - {W_P}^2 \cos^2(\epsilon - \varphi)}{{W_P}^2 \sin^2(\epsilon - \varphi)}\right] \tag{2}$$

$$\frac{\rho_P}{\rho_1} = \frac{\tan \epsilon_P}{\tan(\epsilon - \varphi)_P} \tag{3}$$

$$\frac{p_P}{p_1} = \frac{2\gamma}{\gamma + 1}\left({M_1}^2 \sin^2 \epsilon_P - \frac{\gamma - 1}{2\gamma}\right) \tag{4}$$

$$\Delta s_P = c_v \log_e \frac{p_P}{p_1}\left(\frac{\rho_1}{\rho_P}\right)^{\gamma} \tag{5}$$

In the equations, the subscript P indicates quantities at the point P behind the shock and the subscript 1 indicates free-stream conditions. From two points of the shock A and B (fig. 1), the flow properties at the point C can be determined by means of the characteristic system for rotational flow as is shown in the following discussion.

From A the tangent to the characteristic line of the first family and from B the tangent to the characteristic line of the second family can be drawn (reference 1)

$$\left.\begin{aligned} \left(\frac{dy}{dx}\right)_A &= \tan(\varphi + \beta)_A \\ \left(\frac{dy}{dx}\right)_B &= \tan(\varphi - \beta)_B \end{aligned}\right\} \tag{6}$$

where

$$\sin^2\beta = \frac{\gamma - 1}{2}\left(\frac{1}{W^2} - 1\right)$$

The intersection of the two characteristic lines determines the point C in the first approximation. The properties at C in the first approximation can be determined from the following expressions (reference 1):

$$\frac{W_A - W_C}{W} - \tan\beta(\varphi_A - \varphi_C) - \frac{x_A - x_C}{y} l + \frac{ds}{dn}\frac{1}{\gamma R}\frac{\sin^3\beta}{\cos(\beta + \varphi)}(x_A - x_C) = 0 \qquad (7)$$

$$\frac{W_B - W_C}{W} + \tan\beta(\varphi_B - \varphi_C) - \frac{x_B - x_C}{y} m - \frac{ds}{dn}\frac{1}{\gamma R}\frac{\sin^3\beta}{\cos(\varphi - \beta)}(x_B - x_C) = 0 \qquad (8)$$

where, for two-dimensional phenomena, l and m are zero whereas, for axially symmetrical phenomena,

$$l = \frac{\sin\beta\tan\beta\sin\varphi}{\cos(\varphi + \beta)} \qquad (9)$$

$$m = \frac{\sin\beta\tan\beta\sin\varphi}{\cos(\varphi - \beta)} \qquad (10)$$

In the first approximation the quantities with no index in equation (7) can be assumed equal to the corresponding quantities at the point A and in equation (8) to the quantities at the point B, and for ds/dn the following expression can be used:

$$\frac{ds}{dn} = \frac{\Delta s_A - \Delta s_B}{(x_A - x_C)\frac{\sin\beta_A}{\cos(\beta + \varphi)_A} + (x_B - x_C)\frac{\sin\beta_B}{\cos(\varphi - \beta)_B}} \qquad (11)$$

In this approximation the entropy gradient is assumed to be the same in the zone between B and C as in the zone between A and B. In the process of calculations it is possible that the two known points fall on the same streamline. In this case, the value of ds/dn for the first approximation can be assumed equal to the value of ds/dn at one of the two points. The value of ds/dn can be obtained from the difference in entropy at this point and at a nearby known point on the same characteristic line.

After the properties at the point C in the first approximation are obtained, a second approximation can be obtained in the following way: The value of entropy at the point C is determined from the entropy of the point on the shock that is in the same streamline as the point C. This

point is determined from mass-flow considerations. The flow at C is then determined again by averaging the values of the known terms in equations (6), (7), and (8) between the values at A and C and at B and C, respectively.

For axially symmetrical phenomena the mass flow contained in the stream tube bounded by the two surfaces of revolution having as generatrix the streamline that passes at A and C (m_A) and at B and C (m_B) can be determined from figures 1 and 3:

$$\left.\begin{aligned} m_A &= \int_{y_C}^{y_A} 2\pi y \rho V \frac{\sin\beta}{\sin(\varphi + \beta)}\, dy \\ m_B &= \int_{y_C}^{y_B} -2\pi y \rho V \frac{\sin\beta}{\sin(\varphi - \beta)}\, dy \end{aligned}\right\} \tag{12}$$

The density at any point P can be expressed as a function of the free-stream stagnation density and of the local velocity and entropy in the form

$$\rho_P = \rho_o e^{-\frac{\Delta s}{R}}\left(1 - W_P^2\right)^{\frac{1}{\gamma - 1}} \tag{13}$$

and, therefore, can be calculated at A, B, and C. The velocity at any point along AC and BC can be expressed in the form:

$$V = V_A\left[1 + (y - y_A)\left(\frac{\frac{V_C}{V_A} - 1}{y_C - y_A}\right)\right]$$

Therefore,

$$V = V_A\left[1 + A_1(y - y_A)\right]$$

where

$$A_1 = \frac{\frac{V_C}{V_A} - 1}{y_C - y_A} \tag{14a}$$

Similar procedures may be used to obtain

$$\rho = \rho_A\left[1 + (y - y_A)B_1\right]$$

$$\frac{\sin\beta}{\sin(\varphi + \beta)} = \frac{\sin\beta_A}{\sin(\varphi + \beta)_A}\left[1 + C_1(y - y_A)\right]$$

where

$$B_1 = \frac{\frac{\rho_C}{\rho_A} - 1}{y_C - y_A} \tag{14b}$$

$$C_1 = \left[\frac{\sin\beta_C}{\sin\beta_A}\,\frac{\sin(\varphi + \beta)_A}{\sin(\varphi + \beta)_C} - 1\right]\frac{1}{y_C - y_A} \tag{14c}$$

If, therefore, terms of higher order are neglected,

$$m_A = \pi\rho_A V_A \frac{\sin\beta_A}{\sin(\varphi + \beta)_A}\left(y_A - y_C\right)\left[\left(y_C + y_A\right)\right.$$

$$\left. + \frac{2y_C + y_A}{3}(y_C - y_A)\left(A_1 + B_1 + C_1\right)\right] \tag{15}$$

Similarly,

$$m_B = \pi\rho_B V_B \frac{\sin \beta_B}{\sin(\varphi - \beta)_B}(y_C - y_B)\left[(y_C + y_B) + \frac{2y_C + y_B}{3}(y_C - y_B)\left(A_2 + B_2 + C_2\right)\right] \tag{16}$$

where

$$\left.\begin{aligned} A_2 &= \left(\frac{V_C}{V_B} - 1\right)\frac{1}{y_C - y_B} \\ B_2 &= \left(\frac{\rho_C}{\rho_B} - 1\right)\frac{1}{y_C - y_B} \\ C_2 &= \left[\frac{\sin \beta_C}{\sin \beta_B}\,\frac{\sin(\varphi - \beta)_B}{\sin(\varphi - \beta)_C} - 1\right]\frac{1}{y_C - y_B} \end{aligned}\right\} \tag{17}$$

If a two-dimensional phenomenon is considered,

$$m_A = -\rho_A V_A\left[\frac{\sin \beta_A}{\sin(\varphi + \beta)_A}\,(y_C - y_A) + \frac{\left(y_C - y_A\right)^2}{2}\left(A_1 + B_1 + C_1\right)\right] \tag{18}$$

$$m_B = \rho_B V_B\left[\frac{\sin \beta_B}{\sin(\varphi - \beta)_B}\,\left(y_C - y_B\right) + \frac{\left(y_C - y_B\right)^2}{2}\left(A_2 + B_2 + C_2\right)\right] \tag{19}$$

When the mass flow is determined, the points C' and C'' on the shock along the same streamline that passes at C can be determined for axially symmetrical phenomena from the expressions

$$m_A = \rho_1 V_1 \left(y_A{}^2 - y_C{}^2\right)\pi$$

$$m_B = \rho_1 V_1 \left(y_B{}^2 - y_C{}^2\right)\pi$$

and, for two-dimensional phenomena,

$$m_A = \rho_1 V_1 \left(y_A - y_C\right)$$

$$m_B = \rho_1 V_1 \left(y_B - y_C\right)$$

In the first approximation, C' will be slightly different from C''; therefore, a point C''' will have to be chosen between the two points.

The variation of entropy at C''' can be determined from equation (5), and the entropy at C''' behind the shock is the same as the entropy at C. Now at the point C, the quantities Δs, φ, β, and W are known in the first approximation; therefore, a second approximation can be determined by assuming for the quantities without index in equation (7) the corresponding average values between A and C, and in equation (8) the average values between B and C. In equation (7),

$$\frac{ds}{dn} = \frac{\left(s_A - s_C\right)\left[\cos(\varphi + \beta)_A + \cos(\varphi - \beta)_C\right]}{\left(x_A - x_C\right)\left(\sin \beta_C + \sin \beta_A\right)}$$

and in equation (8),

$$\frac{ds}{dn} = \frac{\left(s_B - s_C\right)\left[\cos(\varphi - \beta)_B + \cos(\varphi - \beta)_C\right]}{\left(x_B - x_C\right)\left(\sin \beta_C + \sin \beta_B\right)}$$

A second approximation can be obtained also for the points C' and C'' and, therefore, for the value of Δs at C. If necessary, a higher approximation can be calculated for Δs. Proceeding in a similar way permits all the characteristic net CDL to be determined and, therefore, the flow properties along CD can be obtained. For inlets the calculation also gives the value of mass flow that goes inside the inlet, because it gives the position of the point E that limits the stream tube that goes inside the inlet.

A check of the precision of the calculations can be obtained from the comparison of the mass flow contained in the stream tube EE' with the mass flow across CD. In addition, the entropy at D must be equal to the entropy at O (fig. 1).

If the tests are performed on a fixed model, a pressure measurement at a point in the zone DF can also be used for control of the precision of the calculations. This control is useful when inlets are considered because the stream tube E''E' (fig. 2) in this case is not known. When the flow field along CD has been determined, the pressure drag along EOD can be determined from the momentum equation, and from the shape of the shock GL and from the characteristic line GD, the pressure along DF and the shape of the streamline DT can be determined. The streamline DT gives the displacement thickness of the boundary layer and of the wake.

CONCLUSIONS

A method for determining pressure drag in axially symmetrical or two-dimensional flow phenomena from shadow or schlieren photographs has been developed. The method uses the characteristic system for rotational flow and can be applied when detached shocks are produced by the body.

The method can be applied to the determination of mass flow and external drag of supersonic inlets with subsonic flow at the entrance of the inlet.

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1. Ferri, Antonio: Application of the Method of Characteristics to Supersonic Rotational Flow. NACA Rep. No. 841, 1946.

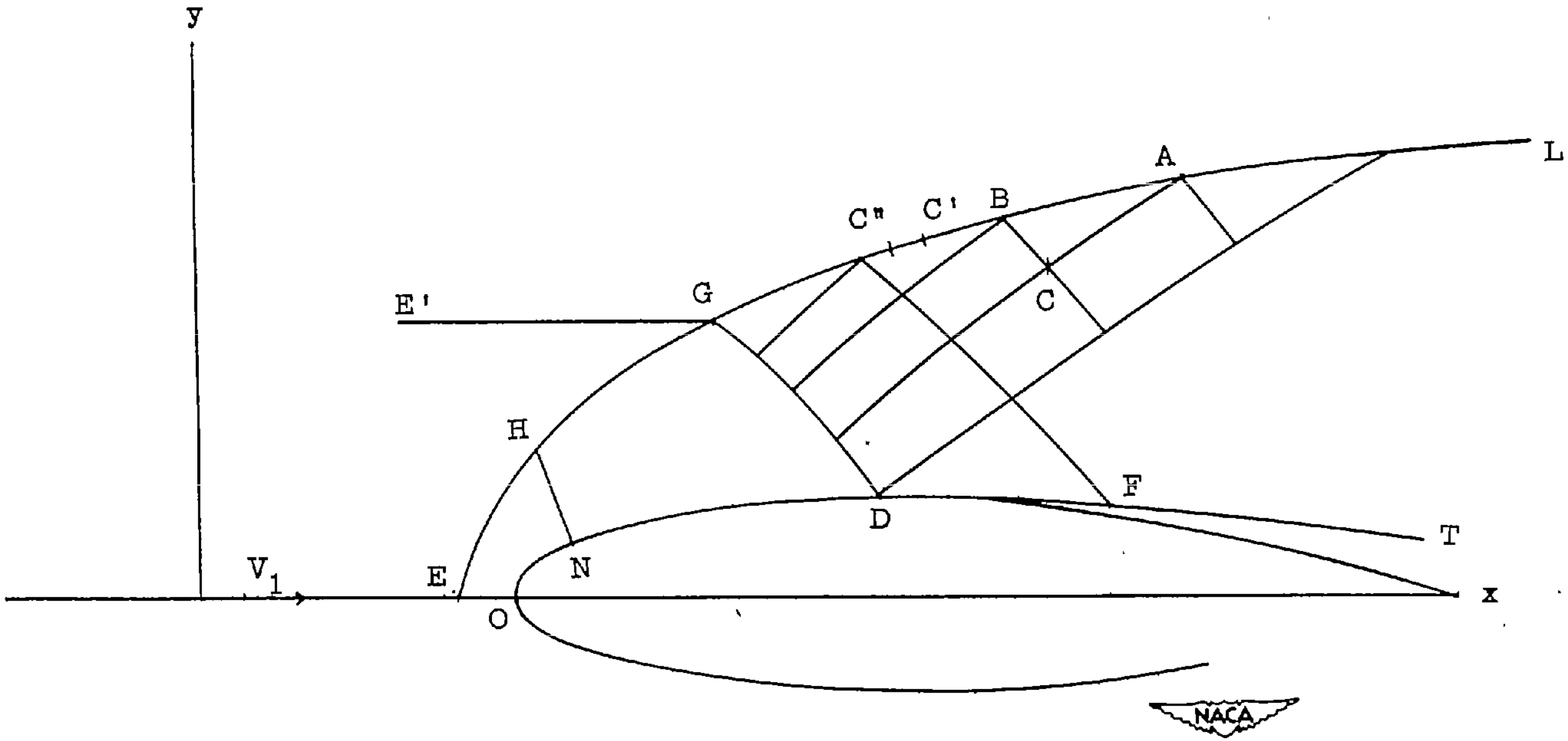

Figure 1.- Scheme of calculations.

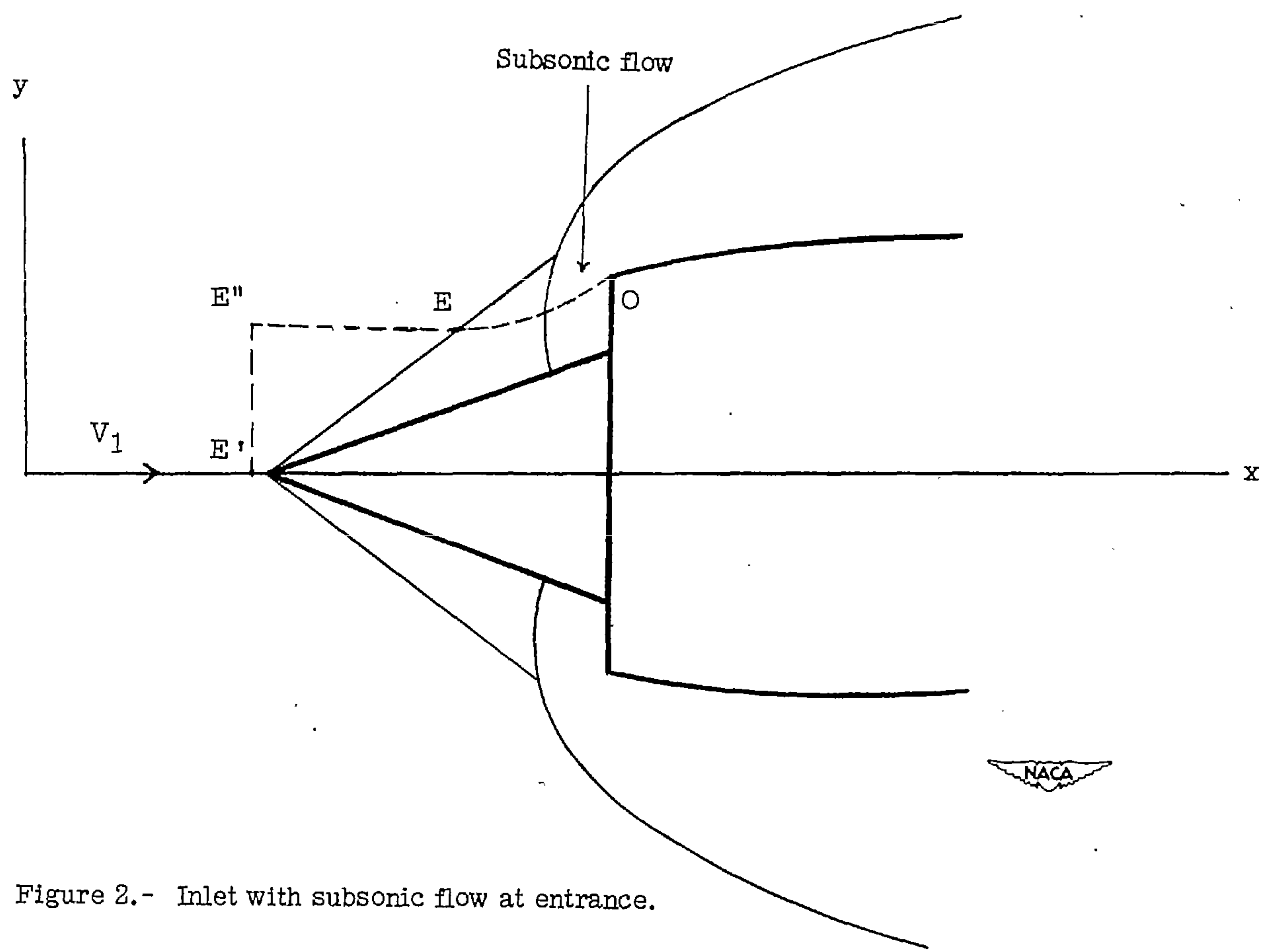

Figure 2.- Inlet with subsonic flow at entrance.

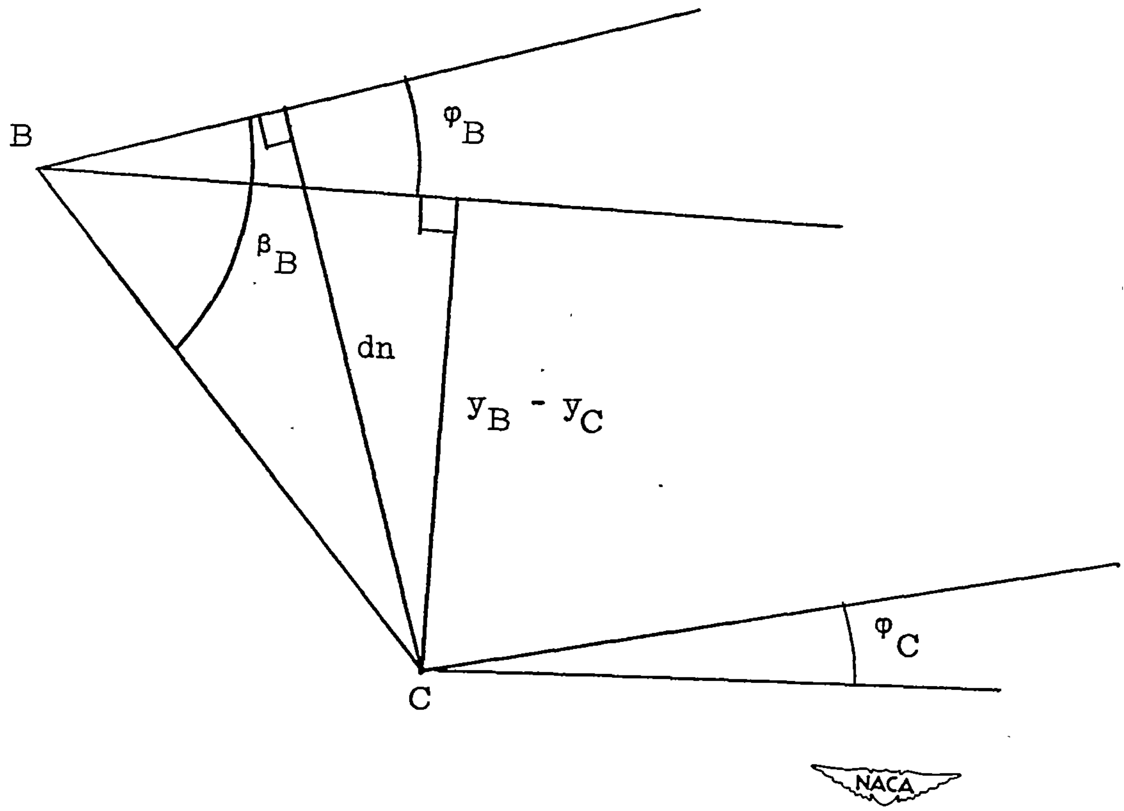

Figure 3.- Calculation of the mass flow.